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Model-independent D^0–$\overline{D}^0$ mixing and CP violation studies with $D^0 \to K^0_S \pi^+ \pi^-$ and $D^0 \to K^0_S K^+ K^-$

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Simulation studies are performed to assess the sensitivity of a model-independent analysis of the flavour-tagged decays $D^0 \to K^0_S \pi^+ \pi^-$ and $D^0 \to K^0_S K^+ K^-$ to mixing and CP violation. The analysis takes as input measurements of the D decay strong-phase parameters that are accessible in quantum-correlated D–$\overline{D}$ pairs produced at the $\psi(3770)$ resonance. It is shown that the model-independent approach is well suited to the very large data sets expected at an upgraded LHCb experiment, or future high luminosity $e^+ e^-$ facility, and that with 100M $K^0_S \pi^+ \pi^-$ decays a statistical precision of around 0.01 and 0.7° is achievable on the CP violation parameters r_{CP} and α_{CP}, respectively. Even with this very large sample the systematic uncertainties associated with the strong-phase parameters will not be limiting, assuming that full use is made of the available $\psi(3770)$ data sets of CLEO-c and BES-III. Furthermore, it is demonstrated that large flavour-tagged samples can themselves be exploited to provide information on the strong-phase parameters, a feature that will be beneficial in the measurement of the CKM angle γ/ϕ_3 with $B^- \to DK^-$ decays.

1 Introduction

In the last few years measurements have been performed at the B-factories and the Tevatron which, when taken together, reveal the presence of mixing in the D^0–$\overline{D}^0$ system [1–6]. Improved sensitivity to this phenomenon will come from the LHCb experiment, which has already published a first study with early data [7]. Interest in the charm sector has been raised still further with the recent announcement of evidence for CP violation in time-integrated $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays* [8,9]. This effect, if confirmed by future measurements, is almost certainly attributable to *direct* CP violation [10]. A natural next step in the experimental programme is to intensify the search for *indirect* CP violation, which manifests itself through mixing-related observables. In the Standard Model (SM) such CP violation is expected to be very small and so any non-zero effect would be indicative of possible New Physics (NP) contributions [11].

A particularly powerful decay mode for flavour-tagged time-dependent D^0–$\overline{D}^0$ studies is the channel $D^0 \to K_S^0\pi^+\pi^-$, where the interference between intermediate resonances provides sensitivity to both the magnitude and sign of the mixing parameters, and allows for probes of indirect CP violation. The analysis of this mode was pioneered by CLEO [12], and higher precision results have been published by Belle [4] and BaBar [5], based on samples of around $0.5\,\mathrm{M}$ and $0.7\,\mathrm{M}$ events respectively. Studies based on larger data sets can soon be expected from LHCb. In time, sample sizes of $\mathcal{O}(10^8)$ events may become available from future experiments, such as Belle-II [13], SuperB [14] and the LHCb upgrade [15, 16]. It is important, therefore, to understand what sensitivity to CP violation can be attained with large data sets of $D^0 \to K_S^0\pi^+\pi^-$ decays, and what challenges such a measurement will be confronted with.

The $D^0 \to K_S^0\pi^+\pi^-$ analysis is performed in the Dalitz space of the three-body final state. Measurement of the mixing and CP violation parameters requires a good understanding of the decay structure within this Dalitz space. The established method is to develop an amplitude model of the D^0 decay on flavour-tagged data, and then exploit this model in an unbinned time-dependent likelihood fit. The drawback to this approach is that the results have a systematic uncertainty that is associated with the assumptions of the model. The size of the uncertainty is difficult to assess, and the degree to which it can be decreased with larger data samples is unknown. An alternative strategy, first proposed in [17], is to perform a binned, model-independent analysis, where the necessary inputs on the D^0 decay come from auxiliary measurements performed on quantum-correlated D–$\overline{D}$ decays at the $\psi(3770)$ resonance. In this case the systematic uncertainties associated with the model are replaced with measurement uncertainties coming from these external inputs. No data analysis has yet been presented which follows this model-independent approach.

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*Unless stated otherwise, the charge-conjugate mode is implicit throughout this article.

Identical considerations apply to the channel $D^0 \to K^0_S K^+ K^-$. Although suppressed by a factor of around six compared to $D^0 \to K^0_S \pi^+ \pi^-$, this decay can be exploited in an identical manner for mixing and CP violation studies. A model-dependent analysis has been performed by BaBar [5], and Belle have studied the time-evolution of the CP-odd component of the decay [6]. The model-independent study of the full Dalitz space is also an appealing analysis option for this decay mode, but one that has not yet been pursued.

This paper presents the results of simulation studies performed to investigate the sensitivity of the model-independent approach to the decays $D^0 \to K^0_S \pi^+ \pi^-$ and $D^0 \to K^0_S K^+ K^-$. The statistical precision of a binned analysis is assessed and compared with that of a model-dependent, unbinned study. The sensitivity is calculated both for the sample sizes that currently exist, and for those which will become available over the coming years. When considering future large data sets, the point at which the analysis will become limited by the existing measurements of the D meson decay properties from the $\psi(3770)$, available from the CLEO-c experiment [18], is determined, together with what improvement in the knowledge of these properties will be required. Various fit strategies are explored which make different use of external inputs. It is shown that other useful results can be obtained from the analysis, aside from the mixing and CP violation parameters.

Model-independent studies of mixing and CP violation with other D^0 decay modes are possible (*e.g.* see Ref. [19]), all of which can benefit from measurements of strong-phase parameters and related properties in $\psi(3770)$ data [20].

2   Formalism

In the neutral D meson system the mass eigenstates, $D_{1,2}$, are related to the flavour eigenstates D^0 and $\overline{D}^0$ as follows:

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle \tag{1}$$

where the coefficients satisfy $|p|^2 + |q|^2 = 1$ and

$$r_{CP}e^{i\alpha_{CP}} \equiv \frac{q}{p}. \tag{2}$$

Indirect CP violation occurs if $r_{CP} \neq 1$ and/or $\alpha_{CP} \neq 0$. Charm mixing is conventionally parameterised by the quantities x_D and y_D, defined as

$$x_D \equiv \frac{M_2 - M_1}{\Gamma}, \qquad y_D \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma} \tag{3}$$

where $M_{1,2}$ and $\Gamma_{1,2}$ are the mass and width of the two neutral D meson mass eigenstates, and Γ the mean decay width of the mass eigenstates. A global analysis of all

current measurements [10] yields the results $x_D = (0.63^{+0.19}_{-0.20})\%$, $y_D = (0.75 \pm 0.12)\%$, $r_{CP} = 0.89^{+0.17}_{-0.15}$ and $\alpha_{CP} = (-10.1^{+9.4}_{-8.8})°$. The values of the latter two parameters are consistent with the null CP violation hypothesis.

The time evolution of neutral D^0 mesons proceeds as follows:

$$|D^0(t)\rangle = \quad g_+(t)|D^0\rangle + \frac{q}{p}g_-(t)|\overline{D}^0\rangle, \tag{4}$$

$$|\overline{D}^0(t)\rangle = \frac{p}{q}g_-(t)|D^0\rangle + \quad g_+(t)|\overline{D}^0\rangle, \tag{5}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-i(M_1 - i\Gamma_1/2)t} \pm e^{-i(M_2 - i\Gamma_2/2)t}\right). \tag{6}$$

Consideration is now given to the specific case in which the D^0 decays to the final state $K_S^0\pi^+\pi^-$ (the formalism presented is identical for $K_S^0 K^+K^-$). The final-state particles are labeled as follows: 1 for K_S^0, 2 for π^+ and 3 for π^-. The Dalitz plot decay amplitude density of $D^0 \to K_S^0\pi^+\pi^-$ is defined as

$$\mathcal{A}_{D^0}(m_{12}^2, m_{13}^2) \equiv a_{12,13}e^{i\delta_{12,13}}, \tag{7}$$

where m_{12}^2 and m_{13}^2 specify coordinates on the $D^0 \to K_S^0\pi^+\pi^-$ Dalitz plot, $a_{12,13}$ is the modulus of the amplitude and $\delta_{12,13}$ is a CP-conserving strong-phase. The decay amplitude density is independent of the time evolution of the D^0 system. Assuming there is no direct CP violation in decays to the final state in question, the equivalent decay amplitude density for $\overline{D}^0$ decays is

$$\mathcal{A}_{\overline{D}^0}(m_{12}^2, m_{13}^2) = \mathcal{A}_{D^0}(m_{13}^2, m_{12}^2) \equiv a_{13,12}e^{i\delta_{13,12}}. \tag{8}$$

The time-dependent probability density in which mixing and indirect CP violation are allowed is

$$\mathcal{P}_{D^0}(m_{12}^2, m_{13}^2, t) = \Gamma\left|g_+(t)\mathcal{A}_{D^0}(m_{12}^2, m_{13}^2) + \frac{q}{p}g_-(t)\mathcal{A}_{\overline{D}^0}(m_{12}^2, m_{13}^2)\right|^2$$

$$= \Gamma|g_+(t)|^2 a_{12,13}^2 + \Gamma\left|\frac{q}{p}\right|^2 |g_-(t)|^2 a_{13,12}^2$$

$$+ \Gamma\,\mathfrak{Re}\left\{\frac{q}{p}g_+^*(t)g_-(t)\, a_{12,13}\, a_{13,12}\, e^{i(-\delta_{12,13}+\delta_{13,12})}\right\}$$

$$+ \Gamma\,\mathfrak{Re}\left\{\left(\frac{q}{p}\right)^* g_+(t)g_-^*(t)\, a_{12,13}\, a_{13,12}\, e^{i(\delta_{12,13}-\delta_{13,12})}\right\}, \tag{9}$$

which, ignoring terms of $\mathcal{O}(x_D^2)$, $\mathcal{O}(x_D y_D)$ and $\mathcal{O}(y_D^2)$, is equal to

$$\mathcal{P}_{D^0}(m_{12}^2, m_{13}^2, t) = \Gamma e^{-\Gamma t}\Big[a_{12,13}^2 + r_{CP}\, a_{12,13}\, a_{13,12}\, \Gamma t\, \big\{y_D \cos(\delta_{12,13} - \delta_{13,12} - \alpha_{CP})$$

$$+ x_D \sin(\delta_{12,13} - \delta_{13,12} - \alpha_{CP})\big\}\Big]. \tag{10}$$

3

Integrating the probability density between times t_a and t_b yields

$$\int_{t_a}^{t_b} \mathcal{P}_{D^0}(m_{12}^2, m_{13}^2, t)\, \mathrm{d}t =$$

$$(e^{-\Gamma t_a} - e^{-\Gamma t_b})a_{12,13}^2 + \left[\Gamma(e^{-\Gamma t_a}t_a - e^{-\Gamma t_b}t_b) + (e^{-\Gamma t_a} - e^{-\Gamma t_b})\right]$$

$$\times\, r_{CP} a_{12,13}\, a_{13,12}\big\{y_D[\cos(\delta_{12,13} - \delta_{13,12})\cos(\alpha_{CP}) + \sin(\delta_{12,13} - \delta_{13,12})\sin(\alpha_{CP})]$$

$$+\, x_D[\sin(\delta_{12,13} - \delta_{13,12})\cos(\alpha_{CP}) - \cos(\delta_{12,13} - \delta_{13,12})\sin(\alpha_{CP})]\big\}. \tag{11}$$

The formalism for the time evolution of the $\overline{D}^0$ decay probability is identical, except r_{CP} is replaced by $1/r_{CP}$ and α_{CP} is replaced by $-\alpha_{CP}$.

If a model is available to describe the variation of amplitude across the Dalitz plot then Eqn. 11 may be used in a maximum likelihood fit, unbinned in Dalitz space, to determine the mixing and CP violation parameters in the analysis of a given data sample. If however a model-independent approach is pursued then it is necessary to partition the Dalitz plane. A total of $2N$ regions are defined, symmetric under the exchange $m_{12}^2 \leftrightarrow m_{13}^2$ and labelled from $-N$ to $+N$ (excluding zero). Then the population in bin i is given by

$$\int_i \int_{t_a}^{t_b} \mathcal{P}_{D^0}(m_{12}^2, m_{13}^2, t)\, \mathrm{d}t\, \mathrm{d}m_{12}^2\, \mathrm{d}m_{13}^2 =$$

$$n\Big\{(e^{-\Gamma t_a} - e^{-\Gamma t_b})T_i + \left[\Gamma(e^{-\Gamma t_a}t_a - e^{-\Gamma t_b}t_b) + (e^{-\Gamma t_a} - e^{-\Gamma t_b})\right]$$

$$\times\, \big\{r_{CP}\sqrt{T_i T_{-i}}(y_D[c_i\cos(\alpha_{CP}) + s_i\sin(\alpha_{CP})] + x_D[s_i\cos(\alpha_{CP}) - c_i\sin(\alpha_{CP})])\big\}\Big\}$$

$$\tag{12}$$

where n is a normalisation factor and T_i, c_i and s_i are defined as follows:

$$T_i \equiv \int_i a_{12,13}^2\, \mathrm{d}m_{12}^2\, \mathrm{d}m_{13}^2\,, \tag{13}$$

$$c_i \equiv \frac{1}{\sqrt{T_i T_{-i}}}\int_i a_{12,13}\, a_{13,12}\cos(\delta_{12,13} - \delta_{13,12})\, \mathrm{d}m_{12}^2\, \mathrm{d}m_{13}^2\,, \tag{14}$$

$$s_i \equiv \frac{1}{\sqrt{T_i T_{-i}}}\int_i a_{12,13}\, a_{13,12}\sin(\delta_{12,13} - \delta_{13,12})\, \mathrm{d}m_{12}^2\, \mathrm{d}m_{13}^2\,. \tag{15}$$

These expressions make use of the relations $c_{-i} = c_i$ and $s_{-i} = -s_i$. In the limit of no mixing or CP violation T_i is proportional to the population in bin i coming from decays of D mesons of a known flavour. The quantity c_i (s_i) is the cosine (sine) of the average strong-phase difference between D^0 and $\overline{D}^0$ decays within bin i, weighted by the decay rate.

As external measurements are available for c_i and s_i (see Sect. 3), and the values of T_i may also be obtained from external data sets, or indeed left as free parameters

4

in the fit (see Sect. 4.1), it is possible to use the observed population of the Dalitz and decay time bins to determine the mixing and CP parameters independent of any model assumptions.

3   Available and future measurements of c_i and s_i

The CLEO collaboration have published results which exploit the quantum-correlated nature of $\psi(3770) \to D\bar{D}$ decays to measure the c_i and s_i coefficients for both $D^0 \to K_S^0 \pi^+ \pi^-$ and $D^0 \to K_S^0 K^+ K^-$ [18]. The study was motivated by the need to provide input for the model-independent measurement of the CKM unitarity angle γ/ϕ_3 in $B^- \to DK^-$ decays [21, 22]. A data set of $0.8\,\text{fb}^{-1}$ is analysed and the parameters extracted from measuring the relative rates of various event classes, for example events containing both a neutral D decay to the signal mode and a neutral D decay to an even or odd CP eigenstate. Binning schemes are defined following the guidance of amplitude models developed at the B-factories with the goal of providing a partitioning which provides good statistical sensitivity in the γ/ϕ_3 analysis. The use of an amplitude model to define the binning introduces no model-dependent bias in the measurement of the c_i and s_i parameters. However if the model does not describe nature well then the statistical power of the binning choice will be diluted with respect to expectation.

In $D^0 \to K_S^0 \pi^+ \pi^-$ decays the coefficients are measured in 8 pairs of Dalitz plane bins. Results are given for four choices of binning: two where the Dalitz space is partitioned into equal intervals of the strong-phase difference, with the variation of the strong-phase difference assumed to follow the models developed by either BaBar [23] ('equal $\Delta\delta_D$ BaBar') or Belle [24] ('equal $\Delta\delta_D$ Belle'); and two where the binning is optimised to yield the best possible sensitivity to γ/ϕ_3 in a low background ('optimal binning') and higher background ('modified optimal binning') environment. For the equal interval strong-phase analysis the typical uncertainties on each c_i (s_i) measurement range from around 0.06 to 0.16 (0.11 to 0.21), depending on the bin.

In $D^0 \to K_S^0 K^+ K^-$ decays the c_i and s_i coefficients are measured in an equal interval strong-phase binning, derived from the model presented in [25]. The lower statistics for this decay restrict the analysis to a fewer number of bins than is the case for $D^0 \to K_S^0 \pi^+ \pi^-$. Results are presented for the Dalitz space partitioned into pairs of 2, 3 and 4 bins. With 4 pairs of bins the precision on c_i varies between 0.07 and 0.36 for c_i, and 0.31 and 0.73 for s_i.

The bin definitions for the equal $\Delta\delta_D$ BaBar $K_S^0 \pi^+ \pi^-$ scheme and that for $K_S^0 K^+ K^-$ with 4 pairs of bins is shown in Fig. 1.

The knowledge of c_i and s_i for both decays can be improved by future measurements. Already the BES III experiment has accumulated around $3\,\text{fb}^{-1}$ at the $\psi(3770)$ resonance and a total integrated luminosity of $5 - 10\,\text{fb}^{-1}$ is foreseen at this collision
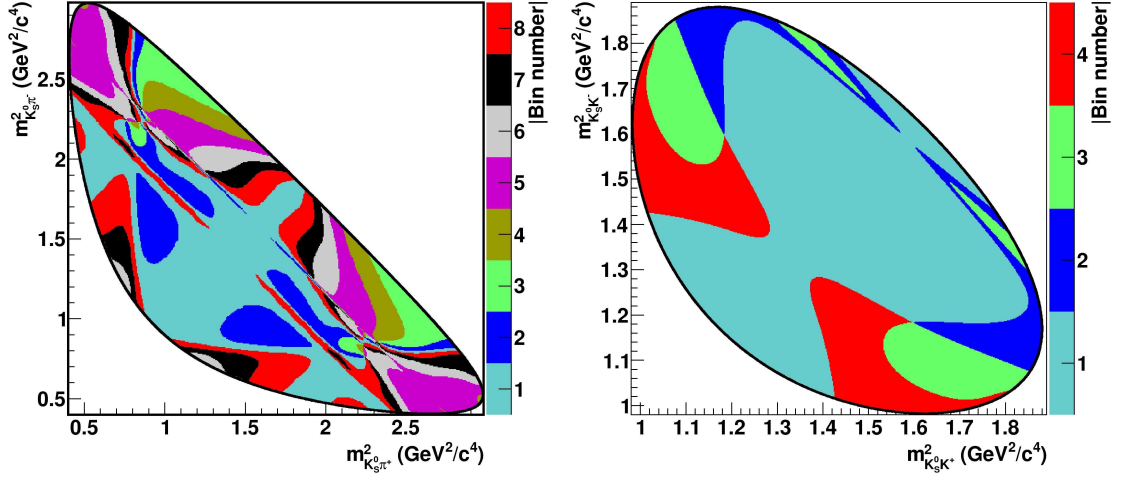
Figure 1: Equal $\Delta\delta_D$ BaBar binning for $K_S^0\pi^+\pi^-$ (left) and for $K_S^0K^+K^-$ (right).

energy [26]. Future facilities, such as a τ-charm factory in Novosibirsk [27] or the SuperB project operating at the charm threshold [14] would provide substantially larger samples of up to 500 fb^{-1}.

4 Fit studies

In order to determine the projected uncertainties on the mixing and CP violation parameters in the model-independent approach a programme of 'toy Monte Carlo' studies is conducted. In each study events are generated in bins of Dalitz space and D^0 decay time according to Eqn. 12. Unless stated otherwise, the binning scheme adopted is the equal $\Delta\delta_D$ BaBar binning with 8 pairs of bins for $K_S^0\pi^+\pi^-$ and the 4 pairs of bins for $K_S^0K^+K^-$. For the results reported below x_D and y_D and the CP violation parameters are set to their current world average values [10]. The number of events in each bin is smeared to account for statistical fluctuations. No background or detector effects are included. A binned maximum likelihood fit is then applied to determine the parameters of interest. Many samples are generated and fit, and in the case of successful convergence the fit parameters are recorded for each simulated experiment. The precision on each parameter of interest is determined by taking the standard deviation of the set of fit values.

In general the conclusions of each study have a dependence on the sample size. Therefore three scenarios are considered:

1. Data sets of a similar magnitude to those that have already been analysed at the B-factory experiments. In the case of $K_S^0\pi^+\pi^-$ this is 0.5M decays, and for

$K_S^0 K^+ K^-$ 0.1M decays.

2. Data sets that may become available at LHCb after several years of operation. Sample sizes of 10M and 2M are assumed for $K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$ respectively.

3. Data sets that may become available at an upgraded LHCb or a future high luminosity $e^+ e^-$ experiment such as Belle-II or SuperB. Sample sizes of 100M and 20M are assumed for $K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$ respectively.

In the case of 2 and 3 the numbers are indicative, and should not be interpreted as official estimates for any of the experiments mentioned.

4.1 Fitting with external input for $(c_i, \ s_i)$

In the first set of studies the c_i and s_i parameters, as measured by CLEO-c, are taken as external input, and the mixing and CP parameters x_D, y_D, r_{CP} and α_{CP} are determined from the fit. The results for 100M $K_S^0 \pi^+ \pi^-$ decays are presented in Table 1.

In one set of experiments, denoted '$(c_i, \ s_i)$ fixed', the values of c_i and s_i are fixed to be the CLEO-c central values in both the generation and fit; in another, denoted '$(c_i, \ s_i)$ smeared', the parameters used in the generation are smeared in a correlated manner according to their measurement uncertainties, but kept fixed to their central values in the fit. Comparison between these two scenarios allows the impact of the $(c_i, \ s_i)$ measurement uncertainties on the results for the mixing and CP violation parameters to be assessed.

It is also possible to use external input for the T_i parameters. Results are available from CLEO-c [18], but significantly more precise measurements can be made with the larger flavour tagged samples that are already available from other experiments. In the studies three scenarios are considered: fixing the values of T_i in the generation and the fit to those measured by CLEO-c ('T_i fixed'); smearing the values in the generation according to the measurement uncertainties that would be achievable with existing B-factory data sets [4], but keeping them fixed to their central values in the fit ('T_i smeared'); fixing the values of T_i in the generation but including them as free parameters in the fit ('T_i floating').

For sample sizes of a few million events and above it is found that the uncertainties on the current measurements of T_i limit the precision with which the mixing and CP parameters can be determined. An estimate of the systematic uncertainty on each parameter coming from this source can be obtained by subtracting in quadrature the results for 'T_i fixed' from those of 'T_i smeared'. This exercise yields errors of approximately 0.216(3)% for x_D, 0.242(3)% for y_D, 0.0483(7) for r_{CP} and 3.65(5)° for

Table 1: Expected uncertainties on mixing and CP parameters for 100M $K_S^0\pi^+\pi^-$ decays, in a variety of fit scenarios. The values in parentheses indicate the precision on each number.

Scenario	$\sigma(x_D)$ (%)	$\sigma(y_D)$ (%)	$\sigma(r_{CP})$	$\sigma(\alpha_{CP})$ (°)
$(c_i,\, s_i)$ fixed, T_i fixed	0.0124(2)	0.0135(2)	0.0116(2)	0.740(10)
$(c_i,\, s_i)$ fixed, T_i smeared	0.2168(29)	0.2424(32)	0.0497(7)	3.721(49)
$(c_i,\, s_i)$ fixed, T_i floating	0.0173(2)	0.0189(2)	0.0118(2)	0.744(10)
$(c_i,\, s_i)$ smeared, T_i floating	0.0757(10)	0.0886(11)	0.0262(3)	1.188(15)

α_{CP}. Improved precision is obtained if the T_i values are left as free parameters in the fit. This approach is therefore considered as baseline for the subsequent discussion.

The statistical uncertainties from the $K_S^0\pi^+\pi^-$ analysis, as estimated from the '$(c_i,\, s_i)$ fixed, T_i floating' ensemble of experiments, for the three sample sizes under consideration are shown in Table 2. In addition are listed the estimated systematic uncertainties in the binned analysis arising from the current knowledge of the $(c_i,\, s_i)$ parameters. These errors are determined by subtracting in quadrature the mean uncertainties of the '$(c_i,\, s_i)$ fixed, T_i fixed' event experiments from those of the corresponding '$(c_i,\, s_i)$ smeared, T_i fixed' studies. Also shown are the statistical and model uncertainties of an unbinned model-dependent Belle analysis [4] conducted with around 0.5M decays.

Table 2: Expected statistical and systematic uncertainties from knowledge of $(c_i,\, s_i)$ in a binned $K_S^0\pi^+\pi^-$ analysis for a variety of sample sizes. The values in parentheses indicate the precision on each number. Also shown are the statistical and model-related systematic uncertainties from an unbinned Belle analysis of 0.5M signal events [4].

Parameter	Binned				Unbinned, 0.5M	
	Stat, 0.5M	Stat, 10M	Stat, 100M	Syst $(c_i,\, s_i)$	Stat	Syst model
$\sigma(x_D)$ (%)	0.251(3)	0.054(1)	0.017(0)	0.076(1)	0.30	$^{+0.09}_{-0.16}$
$\sigma(y_D)$ (%)	0.272(4)	0.061(1)	0.019(0)	0.087(1)	0.25	$^{+0.07}_{-0.08}$
$\sigma(r_{CP})$	0.175(2)	0.037(1)	0.012(0)	0.024(0)	$^{+0.30}_{-0.29}$	0.08
$\sigma(\alpha_{CP})$ (°)	12.46(16)	2.42(3)	0.74(1)	0.88(2)	$^{+16}_{-18}$	$^{+2}_{-4}$

The results in Table 2 indicate that the statistical uncertainties in the binned analysis exhibit an approximate $1/\sqrt{N}$ scaling, as expected. The 0.5M binned statistical uncertainties are similar to those of the unbinned Belle analysis. Indeed, in some cases the binned uncertainties are slightly smaller than those from the unbinned study. It is assumed that this behaviour can be attributed to the idealised nature of

the toy Monte Carlo and the particular properties of the Belle data set. In principle the small loss of information that occurs in a binned study should lead to a slightly reduced statistical precision with respect to an unbinned analysis. Further discussion on this issue can be found in Sect. 4.4.

Comparison of the (c_i, s_i) systematic uncertainties with the statistical uncertainties associated with a data set of a given size allows the potential of the available CLEO-c measurements in the $K_S^0 \pi^+ \pi^-$ mixing and CP violation analysis to be assessed. Of most interest are the CP violation parameters r_{CP} and α_{CP}. Here the (c_i, s_i) uncertainties are smaller than the statistical uncertainties for a 10M event data set. By interpolating the statistical precision it is found that the (c_i, s_i) uncertainty becomes dominant in data sets of around 25M and 75M events, for r_{CP} and α_{CP} respectively. Even in the case of 100M events it would be presumably be possible to make the (c_i, s_i) uncertainty subdominant, merely by repeating the CLEO-c analysis with the 3 fb^{-1} $\psi(3770)$ sample that BES-III now possesses. The current (c_i, s_i) uncertainties are significantly smaller than the assigned model errors in the Belle analysis. Although larger data sets will allow for the $K_S^0 \pi^+ \pi^-$ model to be refined, it is difficult to assess what improvements can be expected in the associated systematic uncertainties.

The precision on the mixing parameters x_D and y_D more rapidly becomes limited by the current knowledge of (c_i, s_i). For 10M events these strong-phase uncertainties are dominant. They are of a similar size to the assigned model uncertainties in the unbinned Belle analysis. Analysis of the quantum-correlated D–$\bar{D}$ data set already available at BES-III would allow for the (c_i, s_i) uncertainty to remain subdominant for $K_S^0 \pi^+ \pi^-$ sample sizes up to around 20M events.

Table 3: Expected statistical and systematic uncertainties from knowledge of (c_i, s_i) in a binned $K_S^0 K^+ K^-$ analysis for a variety of sample sizes. The values in parentheses indicate the precision on each number.

Parameter	Stat, 0.1M	Stat, 2M	Stat, 20M	Syst (c_i, s_i)
$\sigma(x_D)$ (%)	0.807(11)	0.168(2)	0.066(1)	0.330(4)
$\sigma(y_D)$ (%)	0.546(7)	0.102(1)	0.033(0)	0.178(2)
$\sigma(r_{CP})$	0.266(4)	0.073(1)	0.029(0)	0.068(1)
$\sigma(\alpha_{CP})$ (°)	33.55(44)	6.47(9)	2.34(3)	3.88(7)

The statistical uncertainties that are expected for a binned analysis of $K_S^0 K^+ K^-$ data, assuming the four-bin equal strong-phase binning, are shown in Table 3. As expected the precision is worse than in the $K_S^0 \pi^+ \pi^-$ case. The uncertainties on the mixing parameters for the 0.1M scenario are similar to the values of 0.92% for x_D and 0.57% for y_D that are obtained in an unbinned BaBar analysis [5]. The (c_i, s_i)

uncertainties from the CLEO-c analysis are dominant in the measurement of x_D and y_D for the 2M sample, but not so for r_{CP} and α_{CP}. For 20M events the $(c_i,\ s_i)$ uncertainties on r_{CP} and α_{CP} are around twice the size of the expected statistical uncertainties. Exploitation of the already available BES-III $\psi(3770)$ data would allow for the $(c_i,\ s_i)$ uncertainties to be approximately halved.

4.2 Fitting with external input for x_D and y_D

An alternative strategy to that presented above is to consider r_{CP} and α_{CP} as the principal parameters of interest, and to use external measurements to fix x_D and y_D in the fit. In this approach it is natural also to determine $(c_i,\ s_i)$ from the fit, as in most scenarios this may be done with a precision that is similar to or better than that of the CLEO-c analysis. Improved knowledge of $(c_i,\ s_i)$ is useful for the measurement of γ/ϕ_3 using $B^- \to D(K_S^0 h^+ h^-)K^-$ decays.

The free parameters are therefore r_{CP}, α_{CP}, $(c_i,\ s_i)$, T_i, Γ and the overall number of events. The values of x_D and y_D are fixed to their current world-average values [10] in both the generation and the fit. The statistical uncertainties on r_{CP} and α_{CP} and the systematic uncertainties arising from the current knowledge of the mixing parameters, given separately for x_D and y_D, are shown in Table 4. The statistical precision is somewhat worse than that obtained following the procedure presented in Sect. 4.1. The systematic uncertainty from the current knowledge of $(x_D,\ y_D)$ does not dominate the measurement of r_{CP} until the sample size exceeds 10M decays, with the error associated with x_D being the most significant. For α_{CP} the measurement remains statistics-limited up to even larger data sets, making this an attractive and complementary alternative analysis strategy to that discussed in Sect. 4.1.

Table 4: Expected statistical and systematic uncertainties on the CP violation parameters from knowledge of x_D and y_D in a binned $K_S^0\pi^+\pi^-$ analysis for a variety of sample sizes. The values in parentheses indicate the precision on each number.

Parameter	Stat, 0.5M	Stat, 10M	Stat, 100M	Syst (x_D)	Syst (y_D)
$\sigma(r_{CP})$	0.3129(44)	0.0561(8)	0.0170(2)	0.0414(7)	0.0234(5)
$\sigma(\alpha_{CP})$ (°)	27.26(38)	3.35(5)	0.99(1)	0.51(4)	0.29(7)

The uncertainties on c_i and s_i from a sample of 100M $K_S^0\pi^+\pi^-$ events are presented in Table 5. The results depend on how well the input parameters x_D and y_D are known, and so the study is repeated considering a variety of scenarios in which the uncertainty on both parameters is scaled from their current values down by a common factor. Also shown are the measurement uncertainties from the CLEO-c quantum-correlated analysis. It can be seen that even without any improvement in

10

the knowledge of the mixing parameters it is possible to fit c_i and s_i in certain bins with a precision that is similar or better than that of the CLEO-c measurements. If the knowledge of the mixing parameters improves by a factor of three or four in precision, for example through mixing analyses using alternative decay modes or through analysis of independent data sets, then it is possible to reduce significantly the uncertainties on the (c_i, s_i) parameters.

Table 5: Fit uncertainties on c_i and s_i for 100M $K_S^0\pi^+\pi^-$ decays when fixing (x_D, y_D) and assuming a certain knowledge of these parameters with respect to the current precision. Also shown are the current measurement uncertainties on c_i and s_i from CLEO-c [18].

Parameter	Scale factor for current uncertainties on x_D and y_D					CLEO-c
	1	1/2	1/3	1/4	$1/\infty$	
$\sigma(c_1)$	0.125(2)	0.087(1)	0.079(1)	0.076(1)	0.072(1)	0.055
$\sigma(c_2)$	0.085(1)	0.062(1)	0.056(1)	0.053(1)	0.050(1)	0.093
$\sigma(c_3)$	0.068(1)	0.064(1)	0.064(1)	0.062(1)	0.064(1)	0.161
$\sigma(c_4)$	0.108(2)	0.079(1)	0.071(1)	0.068(1)	0.065(1)	0.153
$\sigma(c_5)$	0.159(2)	0.091(1)	0.071(1)	0.060(1)	0.048(1)	0.062
$\sigma(c_6)$	0.114(2)	0.083(1)	0.079(1)	0.076(1)	0.070(1)	0.126
$\sigma(c_7)$	0.065(1)	0.056(1)	0.056(1)	0.056(1)	0.056(1)	0.163
$\sigma(c_8)$	0.103(1)	0.071(1)	0.065(1)	0.062(1)	0.058(1)	0.102
$\sigma(s_1)$	0.102(1)	0.102(1)	0.100(1)	0.098(1)	0.097(1)	0.107
$\sigma(s_2)$	0.123(2)	0.089(1)	0.075(1)	0.067(1)	0.057(1)	0.195
$\sigma(s_3)$	0.225(3)	0.154(2)	0.118(2)	0.099(1)	0.076(1)	0.138
$\sigma(s_4)$	0.169(2)	0.124(2)	0.099(1)	0.091(1)	0.078(1)	0.214
$\sigma(s_5)$	0.055(1)	0.052(1)	0.051(1)	0.051(1)	0.049(1)	0.134
$\sigma(s_6)$	0.219(3)	0.151(2)	0.123(2)	0.105(1)	0.088(1)	0.205
$\sigma(s_7)$	0.236(3)	0.161(2)	0.119(2)	0.094(1)	0.065(1)	0.154
$\sigma(s_8)$	0.170(2)	0.117(2)	0.096(1)	0.084(1)	0.070(1)	0.158

As can be seen from Table 5 the statistical precision is better in some bins compared to others, and the measurement uncertainties from CLEO-c also vary from bin-to-bin. Therefore analysis of sample sizes of $\sim 20\,\mathrm{M}$ decays would already be beneficial for improving knowledge of certain parameters.

A similar analysis performed on $K_S^0 K^+ K^-$ simulated data yields the same qualitative conclusions. Again the precision on the CP violation parameters is inferior to that observed in Sect. 4.1, but it is possible to measure the (c_i, s_i) parameters with a precision that is useful for future γ/ϕ_3 studies.

4.3   Fitting with no external inputs

With a sufficiently large data set it is possible to determine the mixing, CP violation and $(c_i,\ s_i)$ parameters in the fit. In practice this is achieved through a two-stage procedure. In the first stage the parameters of small magnitude in Equation 12, namely x_D, y_D, c_i and s_i, are fixed to their current world-average values and the other parameters are determined. In the second stage the parameters that were fitted in the first stage are fixed, and vice versa.

When this study is performed for ensembles of 100M $K_S^0\pi^+\pi^-$ decays uncertainties of 0.0644(10)%, 0.0790(12)%, 0.0244(4) and 1.762(27)° are obtained for x_D, y_D, r_{CP} and α_{CP} respectively. These results are not competitive with those of the strategies that make use of external inputs. The precision for $(c_i,\ s_i)$ in this study is on average around twice as poor as that obtained when constraining x_D and y_D to lie within their current measurement uncertainties.

4.4   Varying the number of bins

It is interesting to consider the variation in the precision of the fitted parameters if the model-independent analysis is repeated with different numbers of bins. As measurements only exist for 8 pairs of bins in the case of $K_S^0\pi^+\pi^-$ and for 2, 3 and 4 pairs of bins in the case of $K_S^0K^+K^-$, the study instead uses input values of $(c_i,\ s_i)$ and T_i as calculated from the model in Ref. [25], and these are assumed to be known perfectly. The free parameters of the fit are x_D, y_D, r_{CP}, α_{CP}, Γ and the total number of decays. For $K_S^0\pi^+\pi^-$ experiments are performed with 8, 16, 32 and 50 pairs of equal-phase bins, and for $K_S^0K^+K^-$ studies are made with 4, 8, 16 and 32 pairs of bins.

The precision determined for each scenario is plotted in Figs. 2 and 3 for 100M $K_S^0\pi^+\pi^-$ and 20M $K_S^0K^+K^-$ decays. In general the precision improves with the number of bins. In order to quantify this trend the following function has been fit to each variable v:

$$\sigma(v) = a + \frac{b}{N} \tag{16}$$

where N is the number of Dalitz plot bin pairs. The asymptotic parameter a gives the statistical precision that is achievable in the unbinned scenario. The values of a and b for each mixing and CP violation parameter are shown in Table 6.

Although improved precision is in principle achievable by increasing the number of bins from the values for which measurements are currently available, this improvement is in all cases rather modest, even when going to an infinite number of bins ($\sim 10\%$ for most parameters). This indicates that the model-independent analysis with 8 pairs of bins for $K_S^0\pi^+\pi^-$, and 4 pairs of bins for $K_S^0K^+K^-$, has a statistical precision which is only marginally inferior to that of the model-dependent, unbinned analysis. Furthermore there can be no significant improvement in sensitivity by changing the
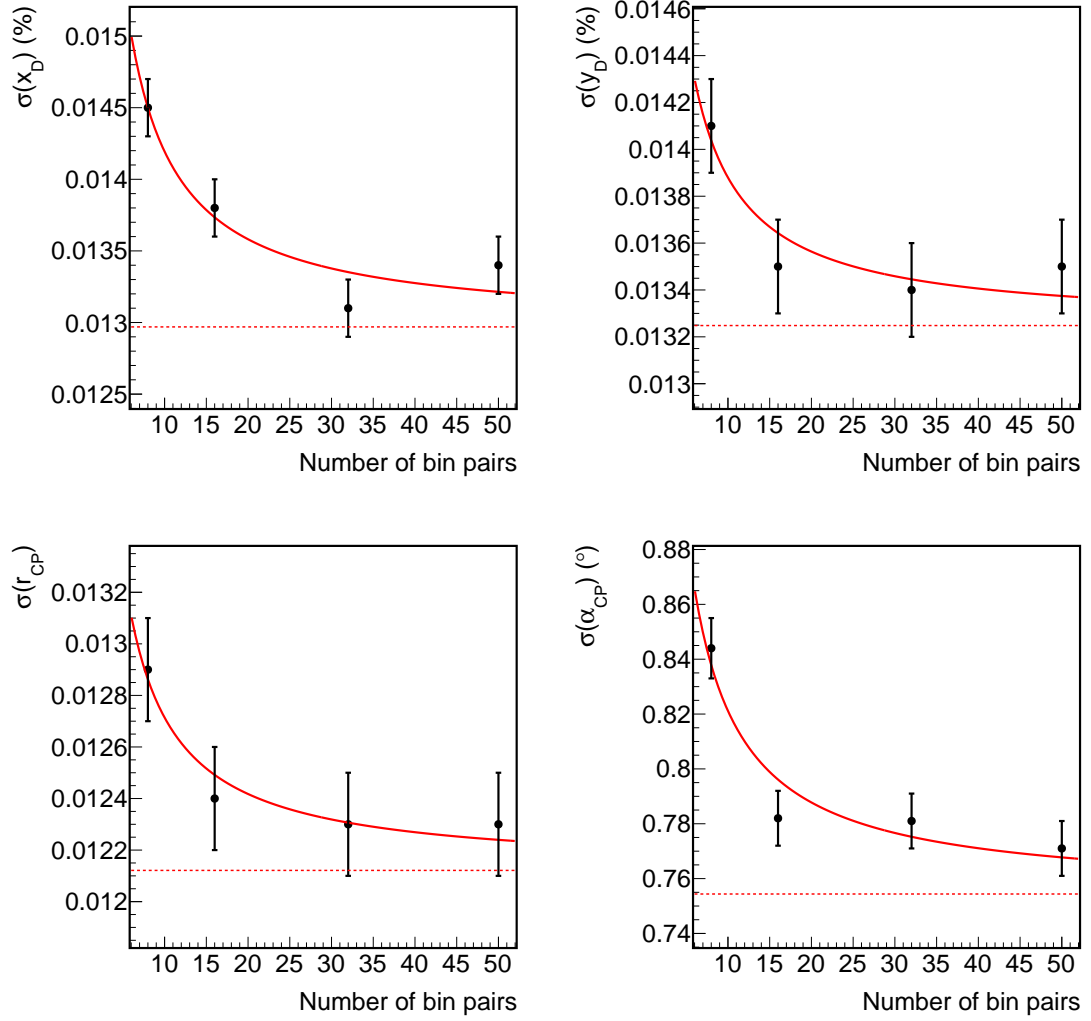
Figure 2: Dependence of the fit uncertainty for the mixing and *CP* violation parameters against number of Dalitz bin pairs for 100M $K_S^0 \pi^+ \pi^-$ decays. The input parameters and fit conditions are described in the text. Superimposed is a fit to Eqn. 16 (solid line) and the asymptotic value of the fit (dotted).
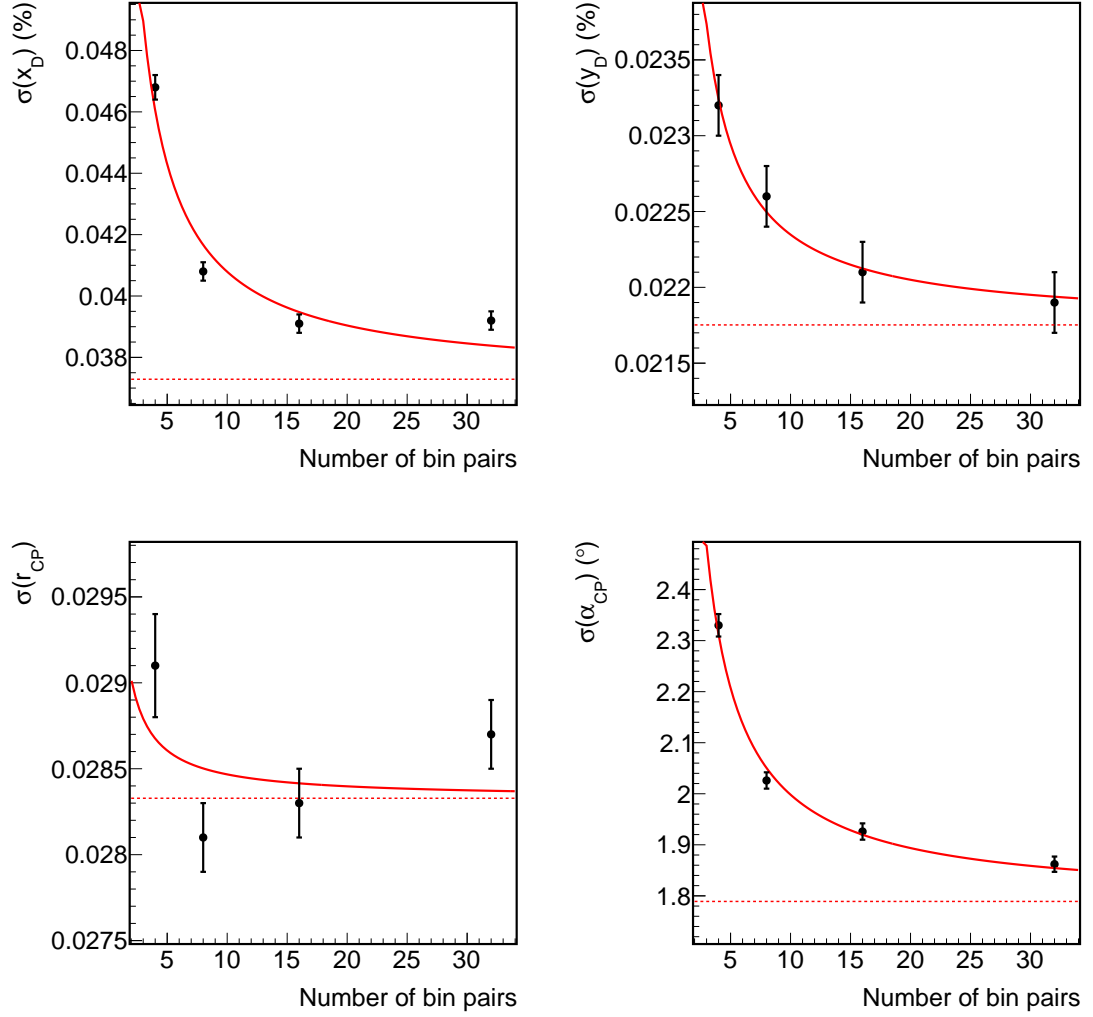
Figure 3: Dependence of the fit uncertainty for the mixing and *CP* violation parameters against number of Dalitz bin pairs for 20M $K_S^0 K^+ K^-$ decays. The input parameters and fit conditions are described in the text. Superimposed is a fit to Eqn. 16 (solid line) and the asymptotic value of the fit (dotted).

Table 6: Results for the parameters a and b, defined in Eqn. 16, that describe the dependence of the precision of the mixing and CP parameters on the number of Dalitz plot bins.

Parameter	$K_S^0 \pi^+ \pi^-$		$K_S^0 K^+ K^-$	
	a	b	a	b
$\sigma(x_D)(\%)$	0.0130(2)	0.0122(25)	0.0373(3)	0.0350(21)
$\sigma(y_D)(\%)$	0.0132(2)	0.0063(25)	0.0218(2)	0.0060(11)
$\sigma(r_{CP})$	0.0121(2)	0.0059(25)	0.0283(2)	0.0014(15)
$\sigma(\alpha_{CP})(°)$	0.754(9)	0.668(130)	1.789(14)	2.091(115)

binning scheme in the analysis from the equal-interval strong-phase binning which has been assumed for the studies presented here. In practice the gain in precision when increasing the number of bins will presumably be even more modest than Figs. 2 and 3 suggest, as the systematic uncertainties associated with the knowledge of (c_i, s_i), not included in this study, will grow as the bin sizes become smaller.

5 Discussion and summary

As already demonstrated by existing measurements, a time-dependent analysis of $K_S^0 h^+ h^-$ decays is a powerful method for studying mixing and to probe for indirect CP violation in the D^0–$\overline{D}^0$ system. Observation of CP violation in charm mixing related observables at a level beyond the very small value predicted in the Standard Model would be a strong indication of New Physics contributions. A binned, model-independent fit is an interesting alternative analysis strategy to the unbinned model-dependent approach which has so far been pursued. An extensive programme of simulation studies has been performed to assess the strengths and limitations of the binned analysis. From these studies three main conclusions emerge:

1. A binned analysis is a very attractive approach for extracting mixing and CP violation information from $K_S^0 h^+ h^-$ decays. By making use of the measurements of the (c_i, s_i) parameters available from CLEO-c it is possible to obtain a statistical precision on x_D, y_D, r_{CP} and α_{CP} that is only marginally inferior to that available from an unbinned analysis. Most importantly, the binned analysis has no systematic error arising from the use of an amplitude model, in contrast to the unbinned approach where this source of uncertainty is unavoidable.

2. The existing CLEO-c measurements of (c_i, s_i), performed with only 0.8 fb^{-1} of $\psi(3770)$ data are adequate for analysing the size of sample that can be expected

at the current LHCb experiment. Looking further forward to the extremely large samples that may be accumulated at the LHCb upgrade or Super-B or Belle-II it is very unlikely that the uncertainties on c_i and s_i will ever become limiting in the measurement of r_{CP} and α_{CP}, which are the principal parameters of interest. Even with 100M $K_S^0 \pi^+ \pi^-$ decays the expected statistical uncertainty on the CP parameters is only around half that of the systematic arising from the CLEO-c measurement errors on c_i and s_i. BES-III already has a $\psi(3770)$ sample that is four times larger than that analysed by CLEO-c, and this is set to grow. Analysis of these data will allow for the (c_i, s_i) systematic uncertainty to become sub-dominant, even for enormous samples of $K_S^0 h^+ h^-$ decays. Plans exist for future projects, such as a Super-B threshold run, or a Novorsibirsk $\tau-$charm factory, that will provide a $\psi(3770)$ data set one-to-two orders of magnitude larger than that accumulated at BES-III. Although welcome for many reasons, these initiatives are not essential for improving the knowledge of the CP violation parameters from the analysis of flavour-tagged $K_S^0 h^+ h^-$ data. On the other hand, the (c_i, s_i) uncertainties will more rapidly become limiting for the mixing parameters x_D and y_D, and here larger quantum-correlated D–$\bar{D}$ samples will be useful.

The bin granularity and partitioning schemes for which the CLEO-c measurements have been performed look well suited for D^0–$\bar{D}^0$ mixing and CP violation studies with all current and future data sets.

3. With very large $K_S^0 h^+ h^-$ samples an alternative fit strategy becomes possible which will benefit the measurement of the unitarity triangle angle γ/ϕ_3 using $B^- \to D(K_S^0 h^+ h^-)K^-$ decays. The induced systematic uncertainty on γ/ϕ_3 arising from the CLEO-c measurements of (c_i, s_i) is estimated to be 2.1° for the equal phase binning [18]. This uncertainty is small compared to the anticipated statistical precision of the current LHCb experiment, but may become limiting at the upgrade, where a measurement error of $\sim 1°$ is expected. A similar precision is foreseen at the Super-B experiment. It is therefore desirable to reduce the uncertainties on (c_i, s_i) by a factor of two or better with respect to the CLEO-c analysis.

Such an improvement may be possible from analysis of the BES-III data alone. However, a complementary measurement of similar precision can be performed using flavour-tagged $K_S^0 h^+ h^-$ data. In this approach external constraints are placed on x_D and y_D and the fit is used to determine (c_i, s_i), as well as r_{CP} and α_{CP}. Assuming 100M $K_S^0 \pi^+ \pi^-$ events, and a knowledge of x_D and y_D which is a factor four times better than at present, then the uncertainties on c_i and s_i can be decreased to around 60% (on average) of their current values. This strategy therefore appears promising, although it requires improved knowledge of x_D and y_D which must come from independent data sets and/or mixing analyses

based on other decay modes.

In conclusion, the model-independent binned fit has many attractive features to recommend it as an analysis strategy for $K_S^0 h^+ h^-$ decays, both with the data sets currently available and the very large samples that will be collected over the coming decade. First results using this approach are eagerly awaited.

Acknowledgments

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